

## MATERIALES EN INGLÉS

*If you know only one language, you live only once.*  
*Proverbio Checo*

*Si hablas sólo un idioma, vives solamente una vez.*  
*Proverbio Checo*

*Those who know no foreign language know nothing of their mother tongue.*  
*Johann Wolfgang von Goethe*

*Aquellos que no hablan un idioma extranjero, no saben nada de su lengua materna.*  
*Johann Wolfgang von Goethe*

La importancia actual del idioma inglés es indiscutible: la tecnología moderna tiene como base el idioma inglés, los materiales que se pueden conseguir en inglés en internet superan en número a los que se pueden conseguir en cualquier otro idioma, el mundo de los negocios habla en inglés, los textos científicos aparecen primero generalmente en inglés y muchos de ellos ni siquiera son traducidos a otros idiomas, las ofertas laborales en el mundo globalizado de hoy privilegian a aquellos aspirantes que hablan por lo menos inglés. Quienes viajan por el mundo siempre encontrarán alguna persona que hable inglés. Además, el inglés es el idioma de dos grandes naciones: Inglaterra y Estados Unidos de América.

Colombia está haciendo un gran esfuerzo por mejorar la enseñanza y el aprendizaje de este idioma ampliando el número de horas y preparando mejor a los encargados de enseñarlo. Ya existen colegios oficiales bilingües.

Colombia Aprendiendo viene ofreciendo varios materiales en inglés desde hace varios años. El Calendario Matemático es traducido mensualmente al idioma inglés y ya hay colegios que lo desarrollan todo el año en este idioma. Igual sucede con las exploraciones. Algunas instituciones educativas, en las que el inglés no es intensivo, desarrollan por lo menos un calendario al año en inglés, apoyándose en el área de idioma extranjero. Consideramos esta una oportunidad apropiada para interrelacionar las dos áreas. La Cartelera Matemática aparece mensualmente en los idiomas español e inglés. Es aconsejable publicarla en ambos idiomas para que los estudiantes se vayan acostumbrando a leerla en las dos lenguas.

A veces publicamos los afiches matemáticos en inglés o partes de ellos en inglés. También hemos publicado nuestra sección de Problemas Rápidos en inglés. En el próximo futuro seguiremos ampliando nuestra oferta de materiales tanto en inglés como en nuestra lengua materna.

# Mathematical Calendar

## MONDAY

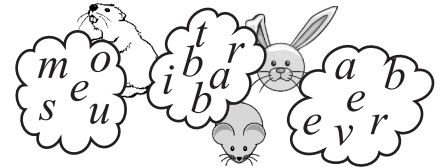


There are magical words you must always be prepared to use:  
Never forget to be thankful,  
to be sorry or to say hallo!

## TUESDAY

## WEDNESDAY

1 Here are hidden the names of three animals that have something in common.



Which are these animals and what do they have in common?

6

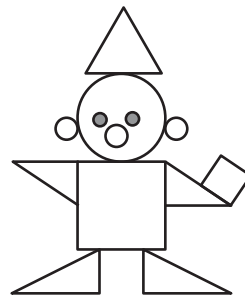
Complete the addition.

$$\begin{array}{r} 4 \quad \square \\ + \quad \square \quad 8 \\ \hline \square \quad \square \quad 5 \end{array}$$

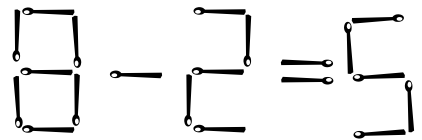
$\square = ?$     $\square = ?$     $\square = ?$

7

Count and complete:  
Triangles: \_\_\_\_\_  
Squares: \_\_\_\_\_  
Circles: \_\_\_\_\_



8

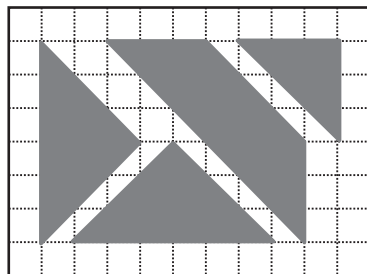


Move a match to make the expression true.

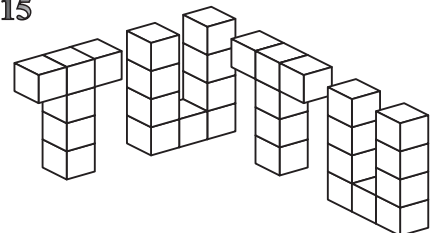
13



14 Make a set of pieces like those on the right, and use them to construct the figure on the left.



15



How many unitary cubes were used to form the word TUTU?

20

Join in ascending order the given points and you will find a beautiful bird.

21

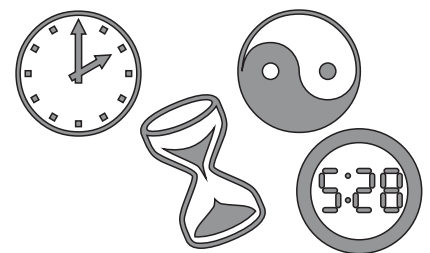
With  
G E N T L E C A R  
1 2 3 4 5 6 7 8 9

You will form the name of a geometric figure. Which one?

9 2 7 4 8 3 1 5 6

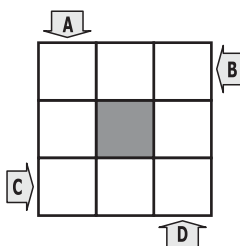
22

Which seems to be the intruder?



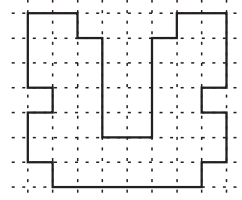
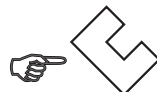
27

- A. One pair
- B. Something used to play baseball
- C. A greasy liquid used for cooking
- D. A room used in scientific experiments



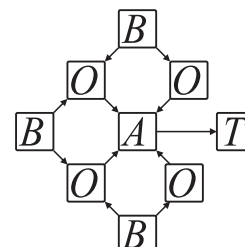
28

Reconstruct the figure on the right with pieces like the one shown.



How many pieces did you use?

29



Read the word *BOAT* in different ways in the arrangement.

# Great Thinkers

## THURSDAY

2

Complete.

7	
-	+
	+
2	4

= 8

## FRIDAY

3

Finish shading so as to obtain the word **GOAL**.

4-5

How many cells were shaded altogether?

9 Who is right?

More paint was used on the figure on the left.

The two figures have the same perimeter.

10

Order the globes bearing in mind the number in them, from least to greatest and you will find the key to success.

----- Which is it?

11-12

In this arrangement all the digits greater than zero are used.

3		2
		5
	9	

→ The sum of these digits is 6.

→ The sum of these digits is 15.

→ The sum of these digits is 24.

↓ ↓

The sum of these digits is 15.

The sum of these digits is 14.

16 Completing words

gra( \_\_ )ach

Only two letters are missing to complete two words, the last two of the first word and the first two of the second word. Which are the two words?

17

Discover the pattern and find the missing numbers.

18-19

Discover the pattern and draw the missing figures.

23 Fill in with green those regions that have two points.

24 Fill in with yellow those regions that have just one point.

What did you discover?

25-26

Join each time four points in order to obtain squares. How many did you manage to get?

30 Help Trosky find Misifú.

Do it in several ways.

Blank space for drawing or writing.

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# Mathematical Calendar

## MONDAY

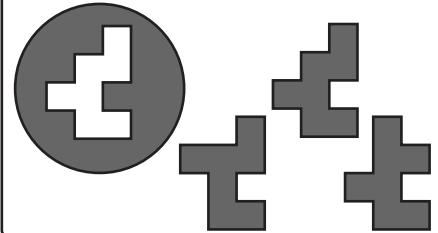


Every thing in its proper place, and every place nice and tidy. That's is how it all should be.

## TUESDAY

## WEDNESDAY

1 Which of the pieces on the right was cut out from the circle on the left?



6 The graph shows the price per kilogram of each of these fruits.

Kathryn buys:  
3 kg. of pineapple  
4 kg. of pears  
2.5 kg of berries.

How much money does Kathryn spend?

7 Use each time two letters in order to form different letters.

b e \_ \_  
b e \_ \_  
b e \_ \_  
b e \_ \_  
b e \_ \_

8 Discover the pattern and complete.

$\begin{array}{ccc} 2 & \times & 9 \\ 4 & \times & 3 \\ \hline & & 5 \end{array}$	$\begin{array}{ccc} 7 & \times & 12 \\ 3 & \times & 2 \\ \hline & & 9 \end{array}$
$\begin{array}{ccc} 4 & \times & 9 \\ ? & \times & 3 \\ \hline ? & & ? \end{array}$	$\begin{array}{ccc} ? & \times & ? \\ 4 & \times & 4 \\ \hline & & 5 \end{array}$

13

14

Make yourself a set of pieces like the ones shown on the left and use them to construct the figure on the right.

15

Hidden here is the name of a person who trained to travel in spacecraft or work in outer space.

**N A S A T U T O R**

Find it!

20

John wishes to distribute 61 marbles in groups of equal quantities, but in any case he wants one marble to be left over. In how many different ways can he do this?

21 True or false

The number of points inside the circle but outside the triangle is half the number of points inside the triangle and inside the circle but outside the circle.

22

Distribute the numbers 1, 3, 6, 7, 8, and 9 in the arrangement in such a way that the sum of the three numbers joined by each segment is 14.

27

All the triangles are equilateral.

If the perimeter of the large triangle is 48, what is the length of the broken line?

28

Moving from left to right, find:

- The greatest possible result.
- The least possible result.

29

Reconstruct the addition.

$H \times H = O$

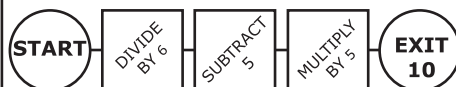
H and V are odd digits.

$$\begin{array}{r} \text{L O V E} \\ + \text{L O V E} \\ \hline \text{H O M E} \end{array}$$

# First Level

## THURSDAY

2

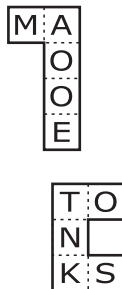


Find the number that must go on the **START** circle.

## FRIDAY

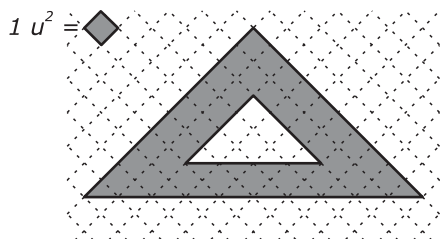
3

Reconstruct the square and you will find a proverb.



9

This is a set square of 45°. What is its area?



16

Discover the pattern and complete:

$$1 = 1$$

$$4 = 1 + 3$$

$$9 = 1 + 3 + 5$$

$$16 = \_ + \_ + \_ + \_$$

$$25 = \_ + \_ + \_ + \_ + \_$$

Write the next two lines of this pattern.

10

Who is right?

Lawrence:

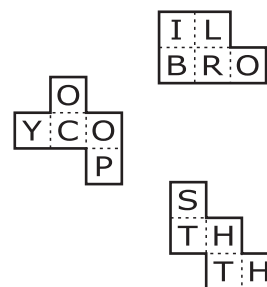
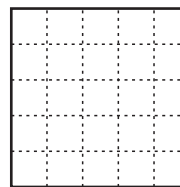
$$9 - 7 + 5 \times 3 - 1 = 20$$

Anselm:

$$9 - 7 + 5 \times 3 - 1 = 16$$

Justify your answer!

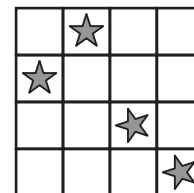
4-5



11-12

Divide the square in four regions of equal shape and size in such a way that there is a star in each region.

Do it in three different ways.

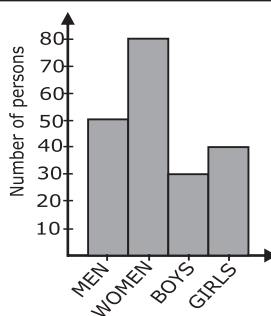


23

The bar diagram represents the number of persons in a certain group. Decide whether the following statements are true or false.

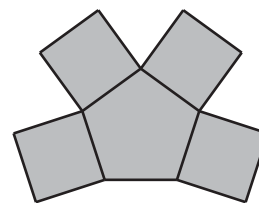
- Both the number of males and females are multiples of 40. \_\_\_
- The number of minors is half the number of adults. \_\_\_

24



25-26

The figure is formed by four squares and a regular pentagon of perimeter 75 cm.



What is the perimeter of the figure?

30

Write the result with letters and discover the hidden word.

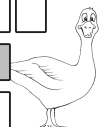
$$(7-3) \times (4-2) = \square \square \square \square$$

$$(6 \times 3 - 2) \div 4 = \square \square \square \square$$

$$(4+7) - 3 \times 3 = \square \square \square \square$$

$$(9 \times 2) \div (2+1) = \square \square \square \square$$

$$(9-2) \times (2-1) = \square \square \square \square$$



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# Mathematical Calendar

## MONDAY

*A smile is the beginning of peace.*

Mother Teresa

## TUESDAY

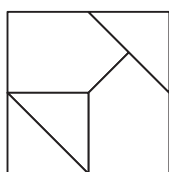
## WEDNESDAY

1

2010 can be written as the sum of fifteen consecutive numbers.

How?

6



With the five figures that form the square, construct the shaded figure.

7

$$\sqrt{1 \times \square + \square} = 4$$

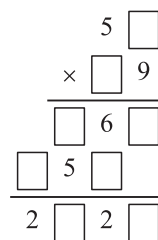
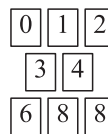
$$\sqrt{\square \times 4 - \square} = 2$$

$$\sqrt{\square \times \square + 6} = 4$$

Inside the three square roots the nine positive digits appear. Complete them!

8

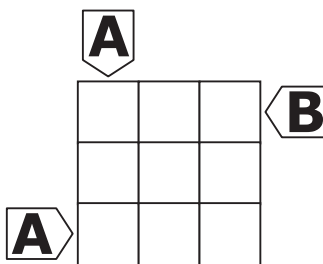
Place the cards adequately in order to complete the multiplication.



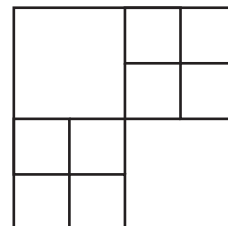
13

A-B

Each cell of the arrangement is either empty or must contain a letter A or a letter B. Complete the arrangement in such a way that in each column and each row each letter appears just once. The letters outside the arrangement indicate which is the first letter to appear in the row or column pointed at.



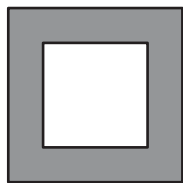
15



How many squares?

20

In the arrangement, the squares are concentric and have integer sides.



If the shaded area is 16 dm<sup>2</sup>, determine the width of the frame.

21

### ANAGRAM

In the following expression the name of a thing you add to make something prettier is hidden.

**"I DO NOT CARE"**

Discover it!

22

Gerald writes the natural numbers one after the other from 1 to 50: 12345678910111213...47484950.

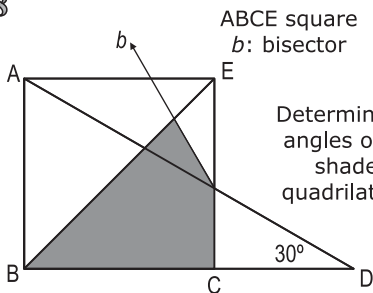
How many digits did Gerald write? What digit occupies the 50<sup>th</sup> position from left to right?

27

By how much does the number 725 increase when

- a digit zero is written on its right?
- a digit 0 is added between the 7 and 2?
- a digit zero is added between 2 and 5?

28



ABCE square  
b: bisector

Determine the angles of the shaded quadrilateral.

29

What digit must be inserted in the empty cell so that the number obtained is divisible by 11.

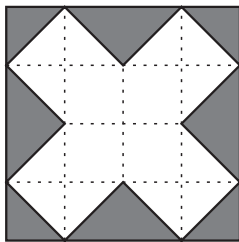


## Second Level

### THURSDAY

2

If the shaded area is 216 cm<sup>2</sup>, find the area of the non-shaded area.



### FRIDAY

3

$$\begin{array}{r} \square 0 \square 0 \mid 5 \square \\ - 1 \square 8 \quad \square 7 \\ \hline 4 \square 0 \end{array} \quad \begin{array}{|c|c|c|} \hline 0 & 2 & 2 \\ \hline 3 & 3 & \\ \hline 6 & 6 & 8 \\ \hline \end{array}$$

Place the cards adequately in order to complete the division.

### FAMILY PROBLEM

4-5

#### Alphabetic

H<E<T<A  
consecutive  
digits

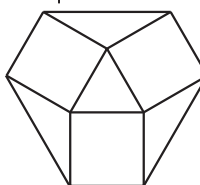
$$\begin{array}{r} C A T \\ + D O G \\ \hline H A T E \end{array}$$

9

A rectangle and a square have integer sides and the sum of their perimeters is 16 dm. If one of the sides of the rectangle measures 3 dm, determine the dimensions of the two quadrilaterals.

10

On the sides of an equilateral triangle three squares are constructed and the necessary vertices of the squares are joined so as to form an hexagon.

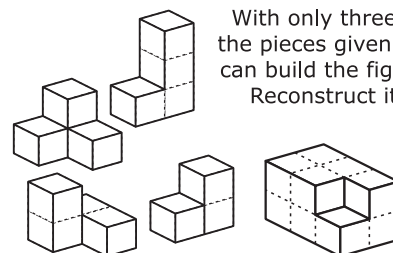


Describe the hexagon and determine its angles.

11-12

#### Logikube

With only three of the pieces given one can build the figure. Reconstruct it!



16

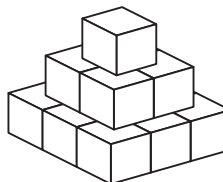
30 can be expressed as the sum of at least three different squares as follows:

$$30 = 5^2 + 2^2 + 1^2$$

Express 42 in a similar way.

17

A pyramid is constructed following the model that is shown.



If 140 blocks were used for its construction, how many levels does the constructed pyramid have?

18-19

#### Intruder

$$\frac{8}{2} + 1 \quad \sqrt{2 \times 8} + 1 \quad 2 \times \sqrt{8 + 1}$$

$$\sqrt{8 + 1} + 2 \quad 8 - (2 + 1)$$

23

#### Greatest-Least

Write in each cell one of the numbers 1, 2, 3 or 4.

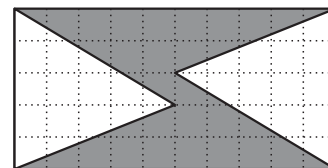
- In each row and each column no number must be repeated.
- The symbols > and < indicate the relation there is between the numbers in the respective cells.

4	>	□	>	□	∇	□
□	<	□	□	1	∧	□
□	∇	2	□	□	∧	□
□	□	□	□	□	□	3

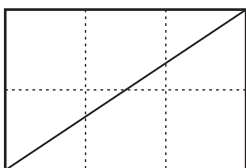
24

25-26

Is it true that the area of the shaded region corresponds to half the area of the rectangle? Justify!



30



In this 3x2 rectangle, the diagonal passes through four unit squares. Through how many unit squares does a diagonal pass in a 9x6 rectangle?

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# Mathematical Calendar

## MONDAY

*Forget injuries, never forget kindnesses.*

*Chinese Proverb*

## TUESDAY

## WEDNESDAY

1

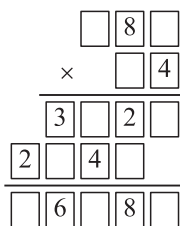
I have two piggy banks, one green and one red. Each day, in the green one, I save two \$ 200 coins and three \$ 500 coins, and in the red one, three \$ 200 coins and two \$ 500 coins. How much money will I have in the red one when I have saved \$ 28,500 in the green piggy bank? How many days until I have that much money saved?

6

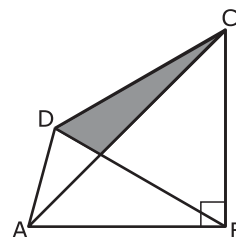
Today, because it is sales day, I can buy a book with a 15% reduction; however I must pay a surcharge of 6%. What is better, that the discount is made first and the surcharge next, or the opposite?

7

Reconstruct the multiplication.



8



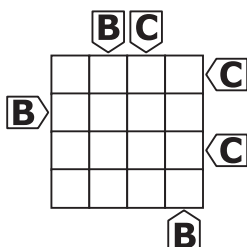
$\triangle ABC$   
isosceles  
rectangle  
 $\triangle BCD$   
equilateral

Determine the angles of the shaded triangle.

13

**A-B-C**

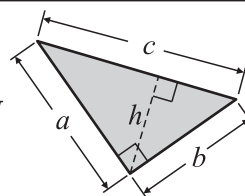
Each cell of the arrangement must be empty or contain one of the letters A, B, or C. Complete the arrangement in such a way that in each column and in each row each letter appears just once. The letters outside the arrangement indicate which is the first letter that appears in the given direction.



14

15

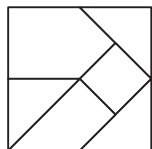
Tito: "In order to find the area of the triangle I must know  $a$  and  $b$ ".



Tato: "In order to find the area of the triangle I must know  $c$  and  $h$ ".

Who is right? Explain!

20



With the six figures that form the square, construct the shaded figure.

21

To go around a circular path, a car takes 2 minutes, a bicycle 6 minutes, and a person walking, 20 minutes. If from the same point, at the same time and in the same direction, a car, a bicycle and an walker depart, after how long will the three meet again?

22

$$\begin{aligned}
 \sqrt{\square \times \square - 2} &= 2 \\
 \sqrt{\square \times \square + \square} &= 7 \\
 \sqrt{4 \times \square - \square} &= 5
 \end{aligned}$$

In the three square roots, the nine positive digits appear. Complete them!

27

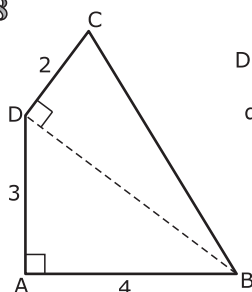
**ANAGRAM**

In the following expression the name of three numbers are hidden.

**"OTHER NEW TOE"**

Find them!

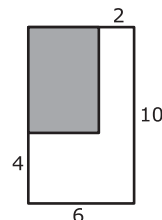
28



Determine the area of the quadrilateral ABCD.

29

The figure represents a rectangular board made of cork.



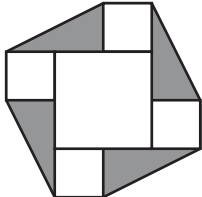
If James throws a dart that falls inside the board, what is the probability that it falls inside the shaded rectangle?



### Third Level

#### THURSDAY

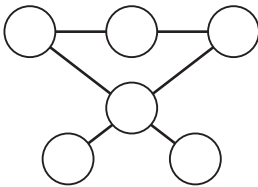
**2** In the octagon, the side of each small square is half the side of the large square.



If the shaded area is  $64 \text{ cm}^2$ , determine the area of the large square and the area of the small square.

#### FRIDAY

**3** Distribute the numbers 5.6, 14.4, 18.4, 27.2, 39.2 and 48 in the arrangement in such a way that the sum of the three numbers joined by each segment is 72.



#### FAMILY PROBLEM

**4-5** **Alphametic**

$$\begin{array}{r}
 \text{O L D} \\
 + \text{M A N} \\
 \hline
 \text{A N D} \\
 \hline
 \text{S E A}
 \end{array}
 \quad
 \begin{array}{l}
 D \times D = L \\
 M \times M = O
 \end{array}$$

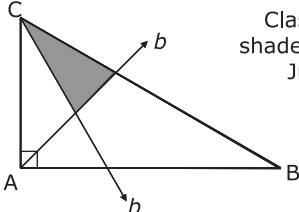
**9** Complete!

If from the sum two of numbers one subtracts their difference the result is equal to:

\_\_\_\_\_

\_\_\_\_\_

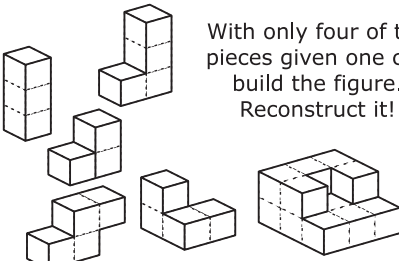
**10**  $\angle ABC = 30^\circ$   
 $b$ : bisector



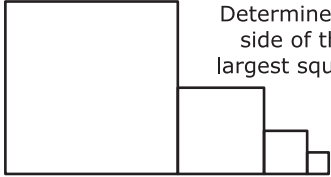
Classify the shaded triangle. Justify!

**11-12** **Logikube**

With only four of the pieces given one can build the figure. Reconstruct it!



**16** The side of each square is half the side of each square immediately before. The total area of the figure is  $340 \text{ dm}^2$ .

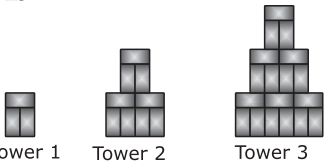


Determine the side of the largest square.

**17** Reconstruct the division.

$$\begin{array}{r}
 \square 7 \square 8 2 \\
 - 1 \square 4 \square \\
 \hline
 1 \square 3 \square \\
 - \square 3 \square 6 \\
 \hline
 1 \square
 \end{array}
 \quad
 \begin{array}{r}
 \square 2 \square \\
 5 \square
 \end{array}$$

**18-19**



Tower 1 has three bricks, tower 2 has nine bricks, tower 3 eighteen bricks. How many bricks are needed to construct the 20th tower?

**23** **Greatest-Least**

Write in each cell one of the numbers 1, 2, 3 or 4.

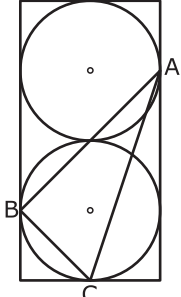
- In each row and each column no number must be repeated.
- The symbols  $>$  and  $<$  indicate the relation there is between the numbers in the respective cells.

**24**

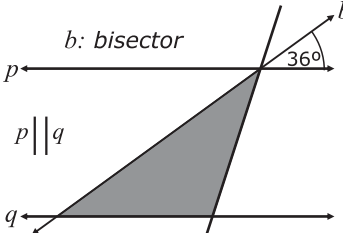
$\square < \square$	$\square$	$\square$
<b>4</b>	$\square$	$\square$
$\square$	$\sphericalangle$	$\square < \square$
$\square$	$\square$	$\square < \square$

**25-26**

Tangent circles of diameter 12 cm each inscribed in a rectangle. A, B, C points of tangency. Determine the area  $\triangle ABC$ .



**30**



$b$ : bisector  
 $\angle = 36^\circ$   
 $p \parallel q$

Classify the shaded triangle. Justify!

Blank space for student response.

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# Mathematical Calendar

## MONDAY

*Failure is only the opportunity to begin again more intelligently.*

Henry Ford

## TUESDAY

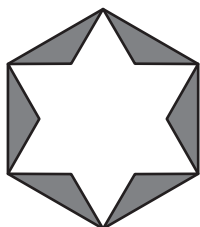
## WEDNESDAY

### 1 Historical

In the middle of a pond there is a bronze lion. Through the eyes, the mouth and the feet streams of water pour out. The stream of water from the right eye would fill on its own the pond in 2 days; the one from the left eye, in 3 days; the one from the feet, in 4 days, and the one in the mouth in 6 days. In how much time would the pond be filled with all 4 streams?

Greek anthology - 500 B.C.

6 Inside a regular hexagon a six-pointed star is drawn.



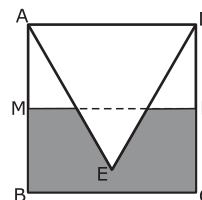
Determine the ratio between the area of the star and the area of the hexagon.

7 In the following expression the name of someone who is trained to save swimmers in danger is hidden.

**"FAIR GLUED"**

Find it!

8  $\triangle ADE$  equilateral  
ABCD square of side  $a$ .

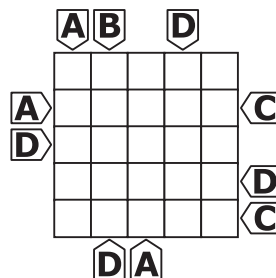


M, N midpoints.

Determine the area of the shaded region in terms of  $a$ .

### 13 A-B-C-D

Each cell of the arrangement must be empty or contain one of the letters A, B, C, or D. Complete the arrangement in such a way that in each column and in each row each letter appears just once. The letters outside the arrangement indicate which is the first letter that appears in the given direction.

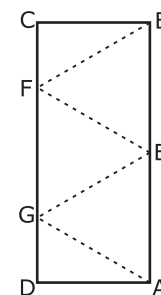


15 At present, the age of Thomas is twice the age Daniel was 10 years ago. Five years ago, the sum of their ages was 39. How old is Thomas at present.

20 One ball is labelled with the number "1", two balls are labelled with "2", three with "3", and so on, until ten balls are labelled with the number "10". The balls are placed in a bag and one ball is removed at a time. How many balls must one remove from the bag so one can be sure to have five balls with the same label?

21 The measures, in cm, of the sides of a triangle, correspond to three consecutive even numbers. The perimeter of the triangle is 18 cm more than the measure of its shortest side. What is the area of this triangle?

22 ABCD rectangle.  
 $\triangle AEG$  and  $\triangle EBF$  equilateral of side 20. Determine the exact value of the diagonal  $\overline{AC}$  and expressed it in the form  $a\sqrt{b}$ .



27 With the six figures that form the square, construct the shaded figure.



28 Reconstruct the multiplication.

$$\begin{array}{r}
 \boxed{A} \boxed{B} \boxed{C} \\
 \times \quad \boxed{D} \boxed{E} \\
 \hline
 \boxed{D} \boxed{F} \boxed{G} \boxed{H} \\
 \boxed{H} \boxed{B} \boxed{I} \boxed{E} \\
 \hline
 \boxed{B} \boxed{I} \boxed{G} \boxed{F} \boxed{H}
 \end{array}$$

H < D  
even consecutive digits

29

$$\sqrt{\square \times \square + \square} = 3$$

$$\sqrt{\square \times 5 - \square} = 1$$

$$\sqrt{\square \times \square + \square} = 6$$

In the three roots the nine positive digits appear. Complete them!

## Fourth Level

### THURSDAY

**2**

$\triangle ABE$  and  $\triangle CDE$  equilateral  
 $\triangle BCE$  isosceles right triangle of area 1.

Determine the exact perimeter of pentagon ABCDE.

### FRIDAY

**3**

A rectangular garden 25 meters wide and 30 meters long has a uniform path of tiles that surrounds it. The area of the garden and the path together is 1254 square meters. What is the width of the path?

### FAMILY PROBLEM

**4-5**

**Alphametic**

$$\begin{array}{r} \text{A U T O} \\ + \text{F O U R} \\ \hline \text{T I R E S} \end{array}$$

$I < U < O$   
consecutive primes

**9**

Reconstruct the division

A	B	C	D	C
–	A	F	G	F
	I	I	B	C
–	I	I	F	D
	A	F		

$C < F$   
odd digits

**10**

Point A has coordinates (2,5). Determine the area and the perimeter of  $\triangle ABC$ .

**11-12**

**Logikube**

With just five of the given pieces you can build the figure. Reconstruct it!

**16**

$\angle BOC = 50^\circ$   
 $\angle AOC = 150^\circ$   
 Determine the angles of  $\triangle ABC$ . Justify!

**17**

Place the adequate exponents so that the expressions are true and equal to 24.

$$2^{\square} - 10^{\square} = 7^{\square} - 5^{\square} = 24$$

$$2^{\square} - 2^{\square} = 3^{\square} - 3^{\square} = 24$$

**18-19**

ABCD square  
 G, F midpoints  
 Determine the ratio of the area of  $\triangle HAB$  to the area of  $\triangle CBE$ .

**23**

**Greatest-Least**

Write in each cell one of the numbers 1, 2, 3 or 4.

- In each row and each column no number must be repeated.
- The symbols  $>$  and  $<$  indicate the relation there is between the numbers in the respective cells.

**24**

<input type="checkbox"/>	<b>1</b>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	$<$	<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	$>$	<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<b>3</b>

**25-26**

The expression  $\frac{2^n - 4^{\sqrt{n}}}{2^{2\sqrt{n}}} = 7$

is true for some digit  $n$ .  
 What is the value of  $n$ ?

**30**

How many pounds of coffee at \$ 4000 a pound must be mixed with 10 pounds of coffee at \$ 6400 a pound in order to obtain a mixture of coffee that costs \$ 5500 a pound?

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# Mathematical Calendar

**MONDAY**

*The difficulty lies not so much in developing new ideas as in escaping from the old ones.*

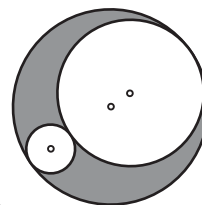
*John Maynard Keynes*

**TUESDAY**

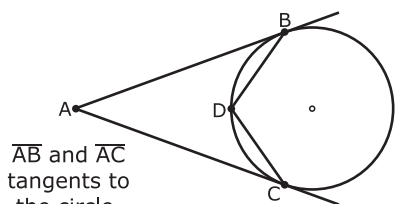
**WEDNESDAY**

**1**

If the diameter of the largest circle is 40 cm, and the ratio between the radii of the smaller circles is 1:3, determine the ratio between the shaded area and the non-shaded area.



**6**



$\overline{AB}$  and  $\overline{AC}$  tangents to the circle.

$\angle CAB = 40^\circ$ .

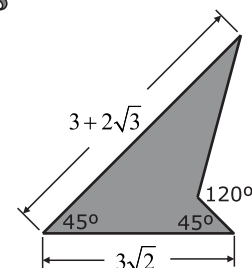
Determine the measure of  $\angle BDC$ .

**7**

Determine the least value of  $n$  for which the following holds:

$$\sum_{k=0}^n \left( \frac{2^{k+1}}{3^k} \right) > \frac{11}{2}$$

**8**

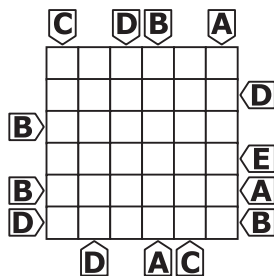


Determine the area and the perimeter of the shaded quadrilateral.

**13**

**A-B-C-D-E**

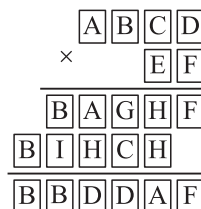
Each cell of the arrangement must be empty or contain one of the letters A, B, C, D or E. Complete the arrangement in such a way that in each column and in each row each letter appears just once. The letters outside the arrangement indicate which is the first letter that appears in the given direction.



**14**

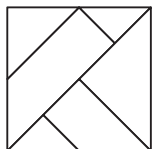
**15**

Reconstruct the multiplication.



F is twice A.

**20**



With the six figures that form the square, construct the shaded figure.

**21**

A farm has enough food to feed 2500 chickens during 75 days. After 29 days, 200 chickens are sold. For how many days will the remaining food last to feed the chickens now in the farm?

**22**

$$\sqrt{\square \times \square + \square} = 2$$

$$\sqrt{\square \times \square - \square} = 4$$

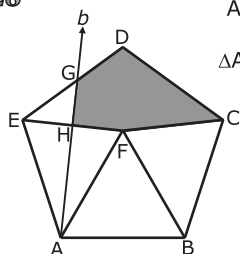
$$\sqrt{\square \times \square - \square} = 1$$

In the three roots the nine positive digits appear. Complete them!

**27**

If  $a+b=p$  and  $ab=q$ , express  $a^3+b^3+a^2b^2$  in terms of  $p$  and  $q$ .

**28**



ABCDE regular pentagon  
 $\triangle ABF$  equilateral  
 $b$ : bisector

Determine the angles of the pentagon CDGHE.

**29**

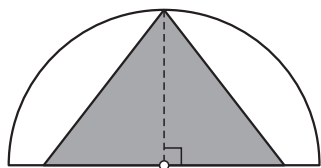
Determine the units digits of the result of the following expression.

$$22^{22} + 33^{33} + 44^{44} + 55^{55}$$

# Fifth Level

## THURSDAY

2



Determine the perimeter of the shaded isosceles triangle in terms of  $\pi$ , if its area is half the area of the semicircle of radius 1 cm.

## FRIDAY

3

Simplify:

$$\frac{2011!}{2010! + 2009!}$$

## FAMILY PROBLEM

4-5

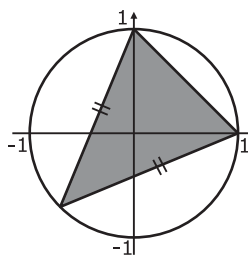
### Alphametic

$$\begin{array}{r} \text{MARS} \\ \text{MOON} \\ + \text{SUN} \\ \hline \text{SPACE} \end{array} \quad \begin{array}{l} N \times N = U \\ N \times U = A \end{array}$$

9

A and B are natural numbers such that neither of them is a multiple of 3. Show that either the sum of A and B or the difference between A and B is a multiple of 3.

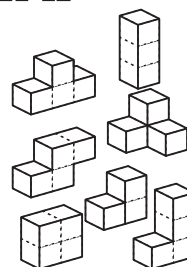
10



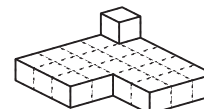
Determine the area of the shaded triangle.

11-12

### Logikube

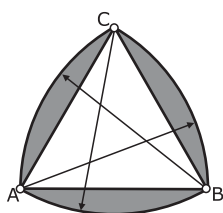


With just six of the pieces shown one can construct the figure. Reconstruct it!



16

$\triangle ABC$  equilateral of side  $a$ .

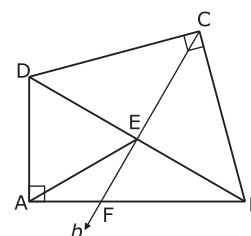


Determine the area of the shaded region.

17

Four friends, Andrew, David, James and Louis play a game of billiards and agree that he who loses must double the amount of money of the other three. They play four games, and each of them loses once: Andrew the first game, David the second, James the third, and Louis the fourth. At that stage, each of them had \$32,000. How much money did each of them have at the beginning?

18-19



$DC = CB$   
 $2AD = DB$   
 $b$ : bisector

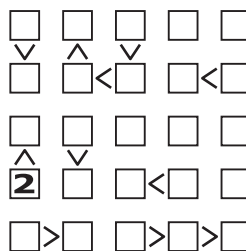
Are the triangles  $\triangle AEF$  and  $\triangle ABE$  similar? Justify!

23

### Greatest-Least

Write in each cell one of the numbers 1, 2, 3, 4 or 5.

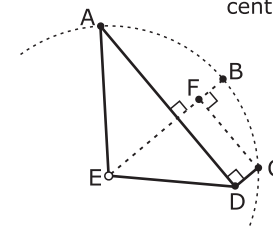
- In each row and each column no number must be repeated.
- The symbols  $>$  and  $<$  indicate the relation there is between the numbers in the respective cells.



24

25-26

A, B, and C are points on the circumference of centre E.



$AE = 10$   
 $AD = 14$   
 $DC = FB = 2$   
 $ED = ?$

30

Reconstruct the division.

$$\begin{array}{r} \boxed{A}\boxed{B}\boxed{C}\boxed{D}\boxed{E}\boxed{F} \quad \boxed{F}\boxed{G}\boxed{H}\boxed{E} \\ - \boxed{A}\boxed{I}\boxed{D}\boxed{B}\boxed{B} \quad \boxed{B}\boxed{D} \\ \hline \boxed{C}\boxed{E}\boxed{C}\boxed{A}\boxed{F} \\ - \boxed{C}\boxed{E}\boxed{C}\boxed{I}\boxed{A} \\ \hline \boxed{A}\boxed{C} \end{array}$$

$A < B$   
consecutive even digits

### Dear Colleague

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◦ Do not use this material without authorization.

*We are what we do  
to change what we are.*

*Eduardo Galeano*

## EXPLORATION

### *Second Level*

### TAXI NUMBERS

For many mathematicians, just the mention of number 1729 brings to memory the following story in which were involved two great mathematicians, G. H. Hardy and Srinivasa Ramanujan:

*«On a certain occasion, when he boarded a tax to travel from London to Putney, Hardy took notice of its number, 1729. He must have thought about it for a short while because when he entered the room where Ramanujan laid ill in bed, barely greeting him, he expressed his deception at such a number. It was, he declared, “a very silly number”, and he added he hoped it was not a bad omen.*

*“No, Hardy, no”, Ramanujan replied. “It is a very interesting number. It is the least number that can be expressed as the sum of two cubes in two different ways.”»*

As a memory of this incident, the least number that is the sum of two cubes in  $n$  different ways is called a Taxi number of order  $n$  and is denoted Taxi ( $n$ ). G. H. Hardy and E. M. Wright proved a theorem that guarantees the existence of Taxi numbers of any order equal or greater than 1. Existence was therefore proved but the search for them is excessively difficult.

Taxi(1) = 2

Taxi(2) = 1729

The first published reference about this property of the number 1729 was found in writings of the XVII century whose author was the French mathematician Bernard Frénicle de Bessy (1604-1607).

Taxi(3) = 87539319

Discovered in 1957 by John Leech (1926-1992) as a result of an extensive computer search.



G. H. Hardy  
1877-1947



Srinivasa Ramanujan  
1887-1920

Taxi(4) = 6963472309248

Discovered in 1991 by a number theory amateur, E. Rosentiel, with the help of computation experts.

Taxi(5) = 48988659276962496

David W. Wilson found the fifth Taxi Number in 1997.

And the search continues, converted into a healthy competition in which amateurs and experts take part, showing among other aspects, one way of challenging the capacities of new computational technologies.

1. Express the number 4104 as the sum of two cubes in two different ways.

2. Ramanujan found the following expression:

$$(3x^2 + 5xy - 5y^2)^3 + (4x^2 - 4xy + 6y^2)^3 + (5x^2 - 5xy - 3y^2)^3 = (6x^2 - 4xy + 4y^2)^3$$

Show that this expression holds for  $x = 2, y = 1$ .

3. Express each of the following numbers as the sum of the least number of cubes:

189

127

100

150

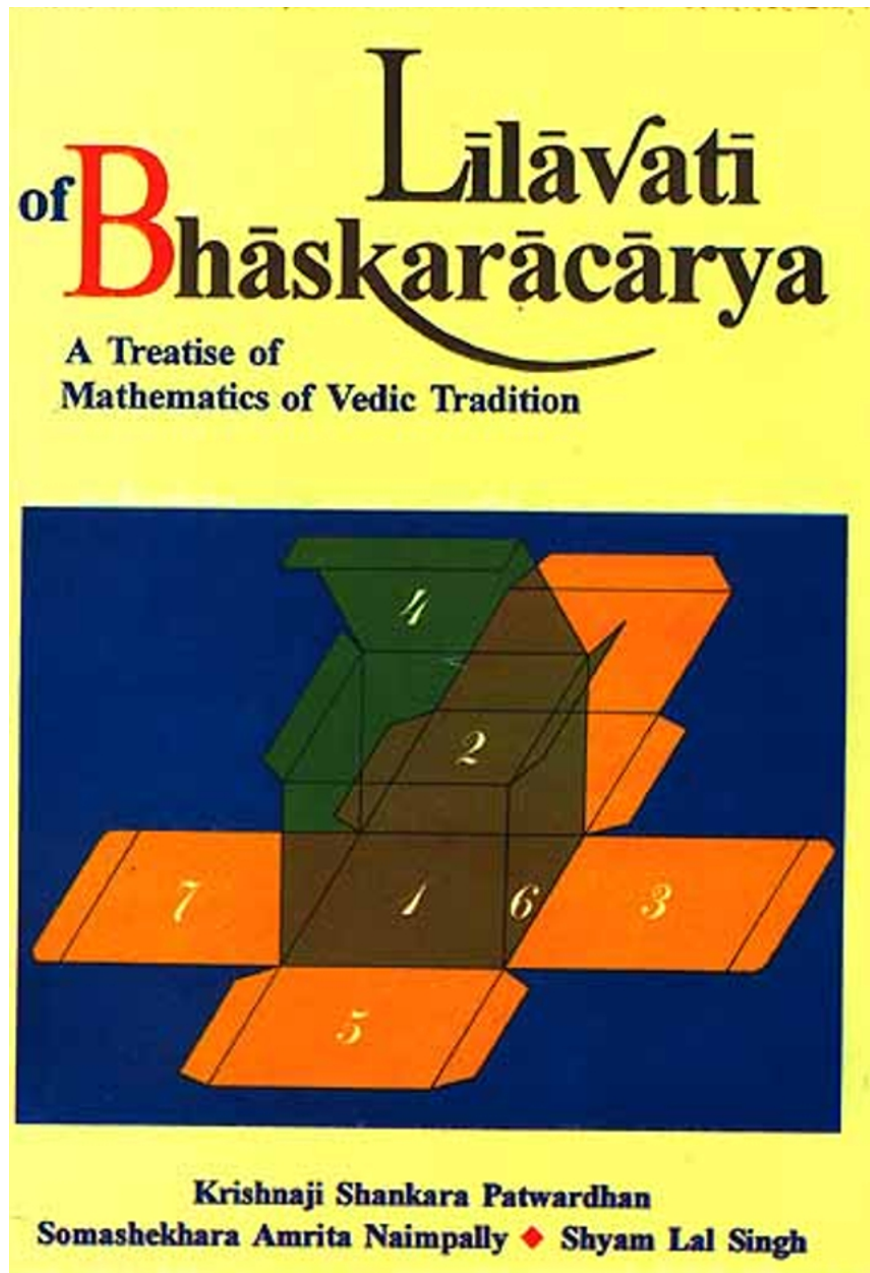
121

*We are what we do  
to change what we are.*

*Eduardo Galeano*

**EXPLORATION**  
*Third Level*

**THE LILAVATI OF BASKARACHARYA**



The Lilavati is a collection of algebra and geometry problems which are completely carried out. Their level of mathematics corresponds to those of high-school algebra, geometry and precalculus. But, in their time, the XII Century, they represented the highest achievements of mathematics in India.



**STANZA CXXXV**

A Donor gave 3 D (drammas) in charity to a Brahmin on the first day. He continued increasing his donation each day by 2 D. If the total amount paid by him equals 360 D, how many days did he give in charity?

Answer: 18

**STANZA LXIV**

O mathematician! Tell me two numbers whose difference is 8 and the difference of whose squares is 400.

Answer: 29 y 21

**STANZA LX**

From a bunch of lotuses,  $\frac{1}{3}$  are offered to Lord Siva,  $\frac{1}{5}$  to Lord Visnu,  $\frac{1}{6}$  to the Sun, and  $\frac{1}{4}$  to the goddess.

The remaining 6 were offered to the guru. Find quickly the number of lotuses in the bunch.

Answer: 120

**STANZA LXXVIII**

To ascertain number ( $x^2$ ), The 18 times of the square root of the number (viz.,  $18x$ ) is added. When one-third of the number (viz.,  $\frac{1}{3}x^2$ ) is added to this sum, the result is 1200. If you know arithmetic well, tell the number.

Answer: 576

**STANZA CCLXXII**

Find quickly the number of different numbers that can be formed with 4, 8, 5, 5, 5. Also find their sum.

Answer.: 20, 1199988

*We are what we do  
to change what we are.*

*Eduardo Galeano*

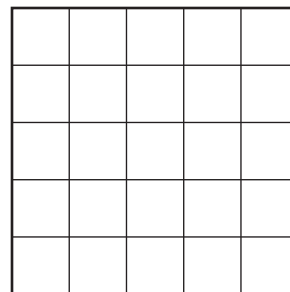
# EXPLORATION

## Fourth Level

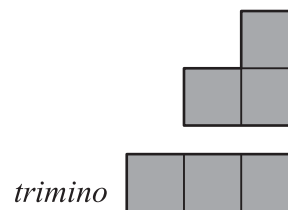
### PATHS

In a  $5 \times 5$  arrangement three cells are shaded.  
The possibilities for shading the three cells correspond to:

- three monominoes
- one domino and one monomino
- one trimino

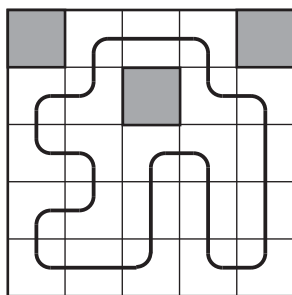


arreglo  $5 \times 5$

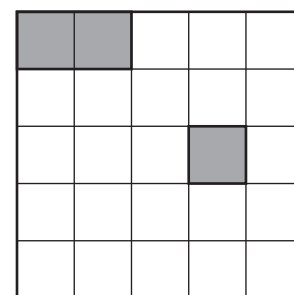


After shading the three cells in the arrangement, the question is whether there is a simple path (without crossings), continuous and closed that passes through each non-shaded cell and such that the path advances only horizontally or vertically.

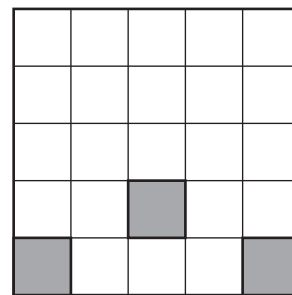
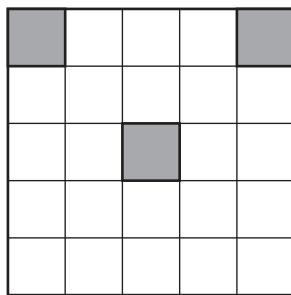
*In this arrangement, three monominoes were used to shade three cells. There are several paths with the given conditions.*



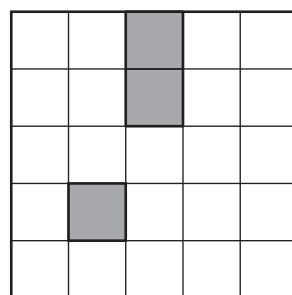
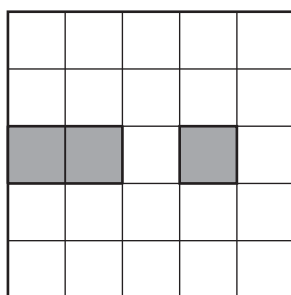
*In this arrangement one domino and one monomino were used to shade three cells. It is impossible to find a path with the given conditions.*



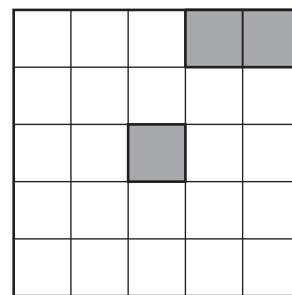
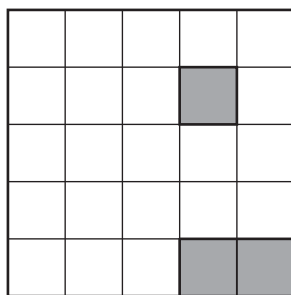
1. Determine in which of these two arrangements there is a path that satisfies the given conditions.



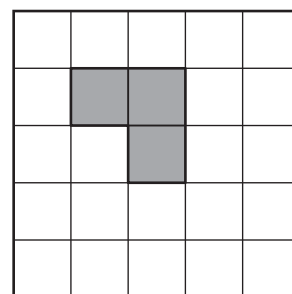
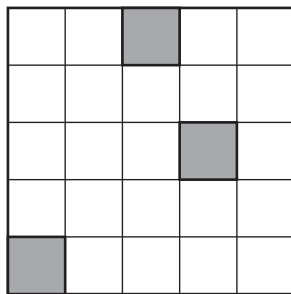
2. Determine in which of these two arrangements there is a path that satisfies the given conditions.



3. In each of these arrangements determine at least two paths that satisfy the given conditions.



4. In each of these arrangements there exists exactly one path that satisfies the given conditions. Determine them!



*We are what we do  
to change what we are.*

*Eduardo Galeano*

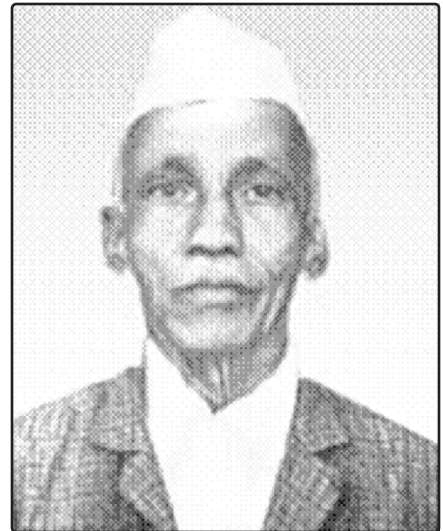
## EXPLORATION

### *Fifth Level*

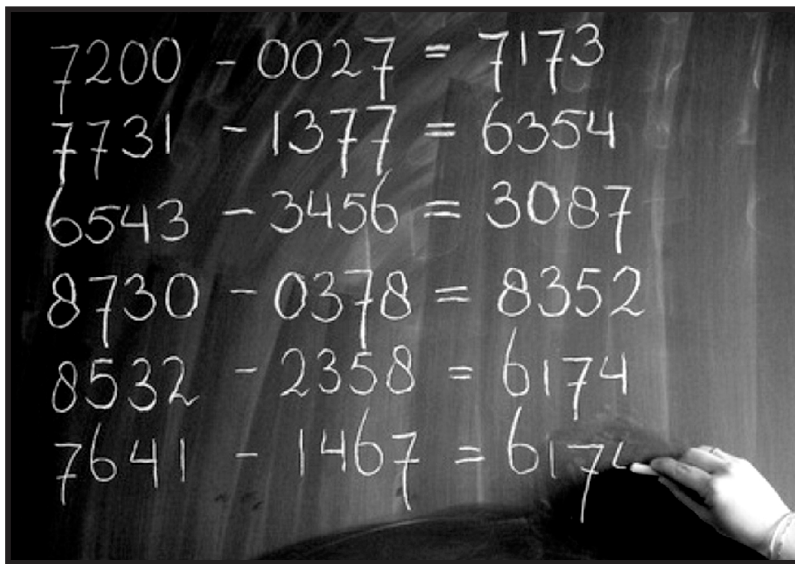
#### KAPREKAR CONSTANT

Dattatreya Ramachandra Kaprekar (1905-1986) was an Indian mathematics professor who was always dedicated to the teaching of mathematics in secondary school.

Kaprekar discovered in 1949 the algorithm that is illustrated below which, when applied to any number with four digits not all the same, eventually arrives at the same number.



6174 is known as a Kaprekar constant.



Explain what Kaprekar's Algorithm is about.

1. Apply Kaprekar's Algorithm to each of the following numbers and determine how many steps are necessary to reach 6174 in each case:

7061, 5814, 8961

2. When one applies Kaprekar's Algorithm to the numbers:

3398, 3399, 3400, 3401, 3402

Which is the one that needs the fewest steps to reach 6174?

Which is the one that needs the most steps?

3. Verify that when you apply Kaprekar's procedure to the following consecutive numbers, they all need the same number of steps in order to reach 6174.

3437, 3438, 3439, 3440

4. Is there a Kaprekar constant for six-digit numbers? Explain!



## Math Posters Primary



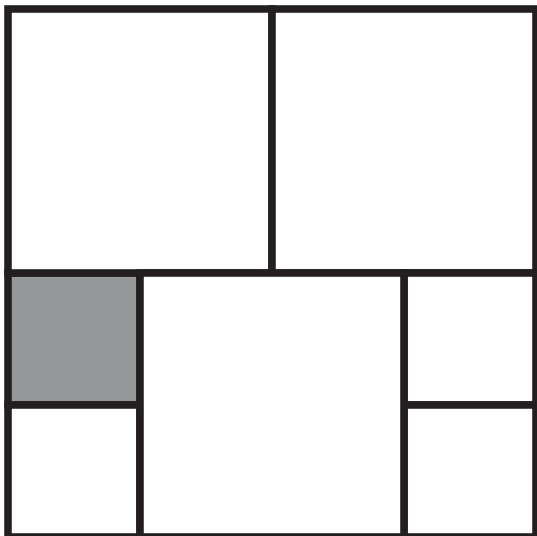
"Just a darn minute!  
- Yesterday you said that  $X$  equals 1!"

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## SHARP EYE

All the figures are squares. By what number must one multiply the perimeter of the shaded square to obtain the perimeter of the largest square?



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## ADDING

Each of the following numbers can be obtained by adding three different numbers all less than 10. How?

8	12	20	
6	10	14	
22	18	16	24

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## PATTERN

There are exactly seven two-digit numbers which belong to the following set.

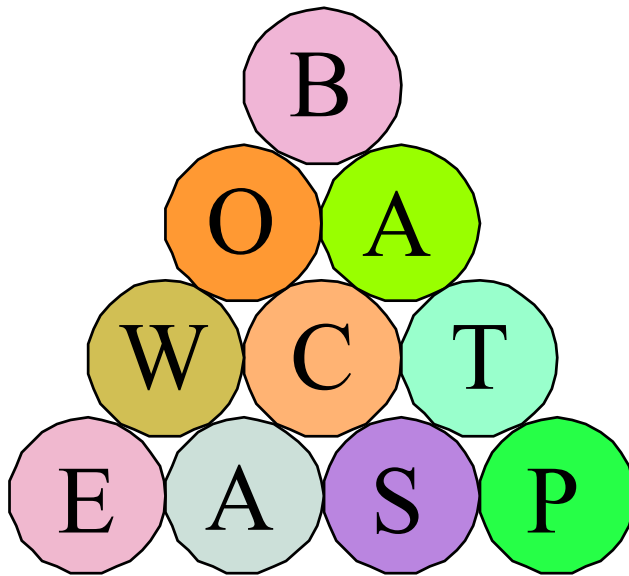
$\{17, 37, 26, 50, 82, \_, \_ \}$

Discover the characteristic that identifies these numbers.

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## SEARCH



Moving from one circle to a neighboring circle, discover the names of five hidden animals.



## CONCENTRATION

If a block weighs  
10 pounds plus  
half a block, how  
much does a block  
and a half weigh?







## Math Posters Secondary

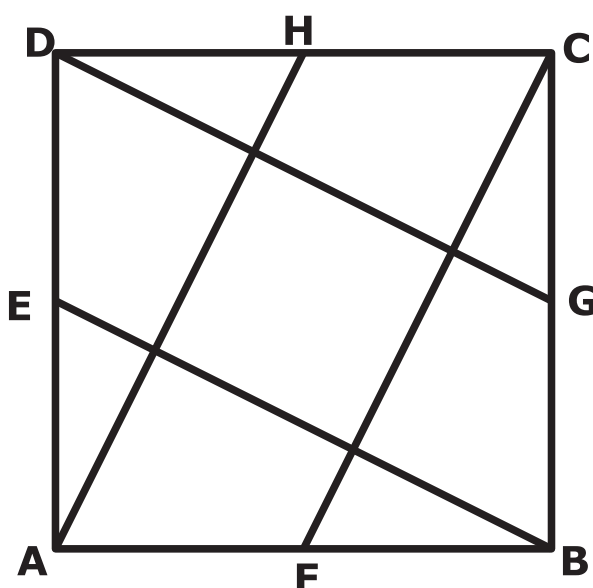
*A business that makes nothing but money  
is a poor kind of business.*

*Henry Ford*

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## COUNTER



ABCD square,  
E, F, G, H midpoints.  
Determine the  
number of trapeziums  
with exactly two right  
angles in the figure.

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# THE OLD PROFESSOR

The old professor began school at the age of 6, and spent 30% of his life getting an education. He then devoted 45% of his life to the teaching of mathematics.

If he has been retired for 14 years now, how old is the professor?



# TRIANGULAR

Each of the words from top to bottom has one letter fewer than the word immediately below, but all its letters already occur in that word. Complete the words with the help of the given descriptions.

- |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | → | One of the colored outer parts of a flower                                      |
| 5 | 4 | 3 | 2 |   | → | When someone comes after the expected time                                      |
| 3 | 2 | 4 |   |   | → | A drink made from the leaves of a shrub that is grown in China, Japan and India |
| 4 | 3 |   |   |   | → | Preposition   |
| 3 |   |   |   |   | → | Consonant   |





## MULTIPLYING

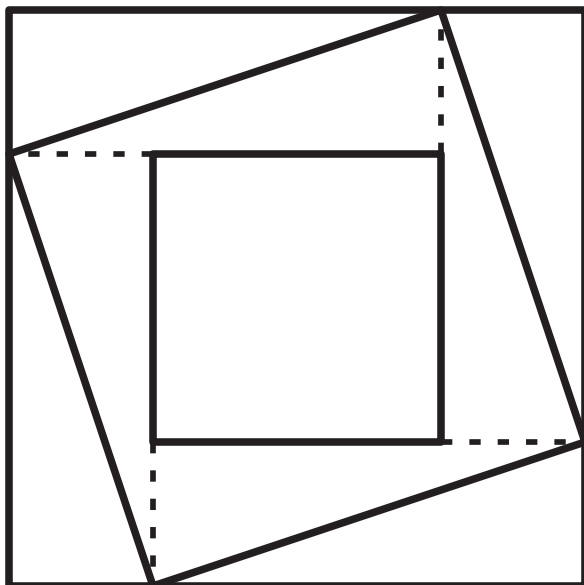
Each of the following numbers can be obtained as the product of three different positive all less than 10. How?

**105    108    112    120    126**

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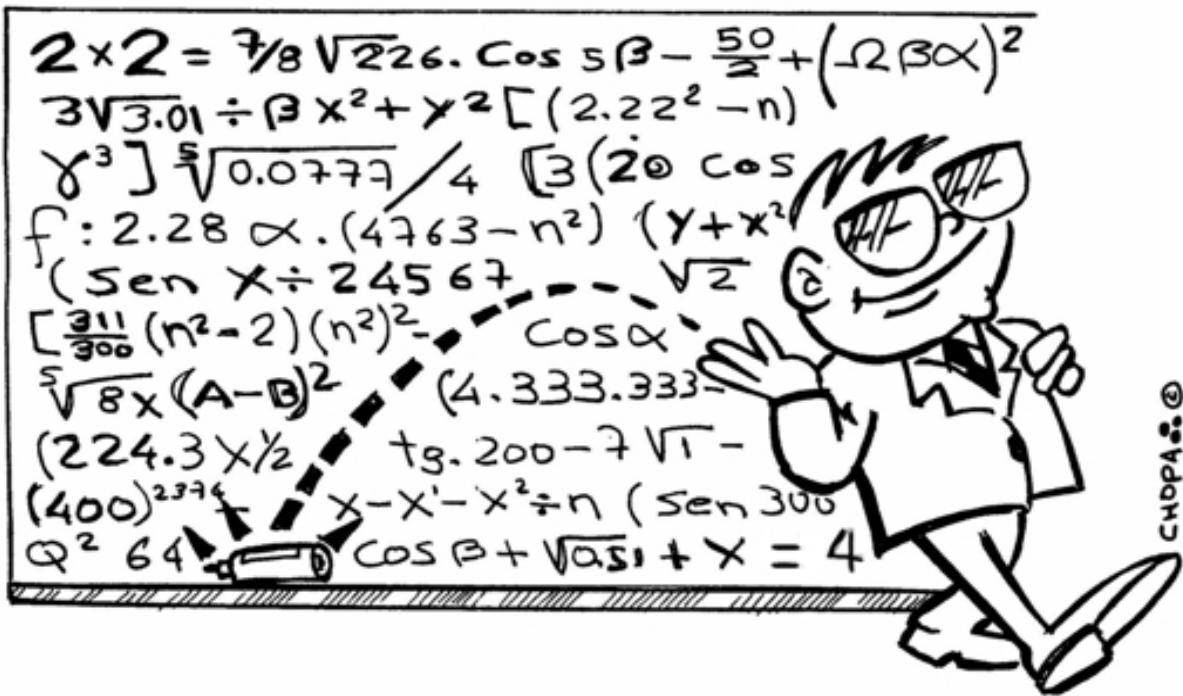


## AREA



In the figure there are three squares. The side of the largest square is 12 cm while the side of the smallest square is 6 cm. Determine the area of the middle square.

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*Maestro: Persona que nos ayuda a resolver problemas que no hubiéramos tenido sin él.*