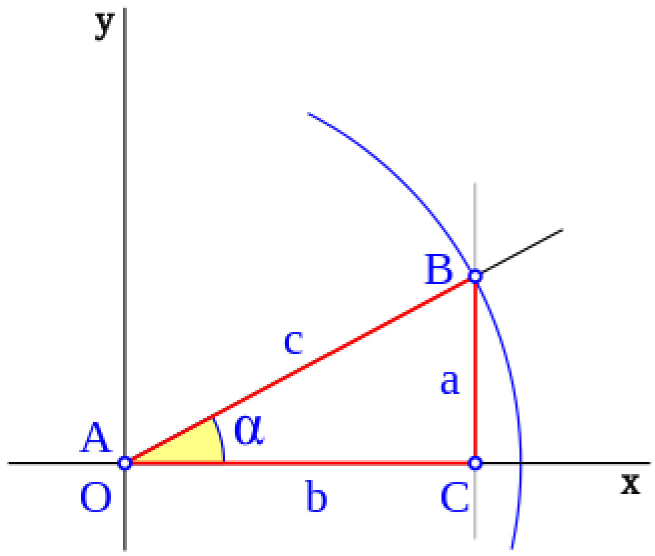
Tangente de la suma y diferencia de dos ángulos

En trigonometría, la tangente (abreviado tan) de un ángulo (en un triángulo rectángulo) se define como la razón entre el cateto opuesto y el adyacente:

$$\tan(\alpha) = \frac{a}{b}$$



O también como la relación entre el seno y el coseno:

$$\tan(\alpha) = \frac{\operatorname{sen}(\alpha)}{\cos(\alpha)}$$

Para trabajar con la tangente de la suma de dos ángulos, lo primero es recordar:

Cos(α+β) =cos α . cos β - sen α . sen β

Sen (α+β) =sen α . cos β – cos α . sen β

$$tg(\alpha+\beta) = \frac{sen(\alpha+\beta)}{cos(\alpha+\beta)}$$

$$tg(\alpha+\beta) = \frac{\text{sen } \alpha \,.\, \cos \beta - \ \cos \alpha \,.\, \text{sen } \beta}{\cos \alpha \,.\, \cos \beta \ - \ \text{sen } \alpha \,.\, \text{sen } \beta}$$

$$tg(\alpha+\beta)=\frac{\frac{\text{sen}\,\alpha\,.\,\cancel{\cos\beta}}{\cos\alpha\,.\,\cancel{\cos\beta}}+\frac{\cancel{\cos\alpha}.\,\text{sen}\beta}{\cancel{\cos\alpha}.\,\cos\beta}}{\frac{\cancel{\cos\alpha\,.\,\cos\beta}}{\cancel{\cos\alpha\,.\,\cos\beta}}-\frac{\text{sen}\,\alpha\,.\,\text{sen}\,\beta}{\cos\alpha\,.\,\cos\beta}}$$

$$tg(\alpha+\beta)=\frac{\frac{\text{sen}\,\alpha}{\cos\alpha}+\frac{\text{sen}\beta}{\cos\beta}}{1-\frac{\text{sen}\,\alpha}{\cos\alpha}.\frac{\text{sen}\beta}{\cos\beta}}$$

$$\boldsymbol{tg(\alpha+\beta)=\frac{\text{tg}\,\alpha+\text{tg}\,\beta}{1-\ \text{tg}\alpha\,.\,\text{tg}\beta}}$$

Encontrar: tg75°

$$tg(\alpha+\beta)=\frac{\text{tg}\,\alpha+\text{tg}\,\beta}{1-\ \text{tg}\alpha\,.\,\text{tg}\beta}$$

tg(45°+30°)= $\frac{\text{tg}45°\ +\text{tg}\ 30°}{1-\text{tg}45°\,.\ \text{tg}30°}$

tg(45°+30°)= $\frac{1+\sqrt{3}/3}{1-1.\sqrt{3}/3}$

tg(45°+30°)=3,73

De forma análoga se llega a la conclusión que :

$$tg(\alpha-\beta)=\frac{\text{tg}\,\alpha-\text{tg}\,\beta}{1+\ \text{tg}\alpha\,.\,\text{tg}\beta}$$

EJERCICIOS RESUELTOS

1. tg(225° + 30°)

Solución:

$$tg(\alpha + \beta) = \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg}\alpha \, . \, \text{tg}\beta}$$

$$tg(225° + 30°) = \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \, . \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{\cancel{3}}}{\frac{3-\sqrt{3}}{\cancel{3}}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{3 + \sqrt{3}}{3 - \sqrt{3}} . \frac{3 + \sqrt{3}}{3 + \sqrt{3}} = \frac{(3 + \sqrt{3})^2}{3^2 - (\sqrt{3})^2}$$

$$= \frac{9 + 6\sqrt{3} + (\sqrt{3})^2}{9 - 3} = \frac{12 + 6\sqrt{3}}{6}$$

$$= \frac{\cancel{6}(2 + \sqrt{3})}{\cancel{6}} = 2 + \sqrt{3}$$

$$\boldsymbol{tg(225° + 30°) = 2 + \sqrt{3}}$$

2. tg(30°+45°)

Solución:

$$tg(\alpha + \beta) = \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg}\alpha \, . \, \text{tg}\beta}$$

$$tg(30° + 45°) = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \, . \, 1} = \frac{\frac{\sqrt{3}+3}{\cancel{3}}}{\frac{3-\sqrt{3}}{\cancel{3}}}$$

$$= \frac{\sqrt{3} + 3}{3 - \sqrt{3}} . \frac{3 + \sqrt{3}}{3 + \sqrt{3}} = \frac{(\sqrt{3} + 3)^2}{3^2 - (\sqrt[\cancel{2}]{3})^{\cancel{2}}}$$

$$= \frac{\sqrt[\cancel{2}]{3}^{\cancel{2}} + 6\sqrt{3} + 3}{6} = \frac{3 + 6\sqrt{3} + 9}{6}$$

$$= \frac{12 + 6\sqrt{3}}{6} = \frac{\cancel{6}(2 + \sqrt{3})}{\cancel{6}}$$

$$\boldsymbol{tg(30° + 45°) = 2 + \sqrt{3}}$$

3. tg(30°+60°)

Solución:

$$tg(\alpha + \beta) = \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg}\alpha \, . \, \text{tg}\beta}$$

$$tg(30° + 60°) = \frac{\frac{\sqrt{3}}{3} + \sqrt{3}}{1 - \frac{\sqrt{3}}{3} . \sqrt{3}} = \frac{\frac{4\sqrt{3}}{3}}{1 - \frac{3}{3}} = \frac{\frac{4\sqrt{3}}{3}}{0} = \pm\infty$$

$$\boldsymbol{tg(30° + 60°) = \pm \, \infty}$$

4. tg($60° + 45°$)

Solución:

$$tg(\alpha+\beta) = \frac{\text{tg}\,\alpha + \text{tg}\,\beta}{1 - \text{tg}\alpha\,.\,\text{tg}\beta}$$

$$tg(\alpha+\beta) = \frac{\sqrt{3}+1}{1-\sqrt{3}\,.\,1} = \frac{\sqrt{3}+1}{1-\sqrt{3}}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}}\,.\,\frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{(\sqrt{3}+1)^2}{1-3}$$

$$= \frac{3+2\sqrt{3}+1}{-2} = \frac{4+2\sqrt{3}}{-2}$$

$$= -2-\sqrt{3}$$

$$\boldsymbol{tg(\alpha+\beta) = -2-\sqrt{3}}$$

5. tg($150° + 30°$)

Solución:

$$tg(\alpha+\beta) = \frac{\text{tg}\,\alpha + \text{tg}\,\beta}{1 - \text{tg}\alpha\,.\,\text{tg}\beta}$$

$$tg(\alpha+\beta) = \frac{-\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}\,.\,\frac{\sqrt{3}}{3}} = \frac{0}{1-\frac{3}{9}} = \frac{0}{1-\frac{3}{9}} = \frac{0}{\frac{6}{9}}$$

$$= 0$$

$$\boldsymbol{tg(\alpha+\beta) = 0}$$

6. Tg(180° - $30°$)

Solución:

$$tg(180° - 30°) = \frac{0-\frac{\sqrt{3}}{3}}{1+0.\frac{\sqrt{3}}{3}} = \frac{-\frac{\sqrt{3}}{3}}{1} = -\frac{\sqrt{3}}{3}$$

$$\boldsymbol{tg(180° - 30°) = -\frac{\sqrt{3}}{3}}$$

7. tg(180° - $10°$)

Solución:

$$tg(180° - 10°) = \frac{0-0{,}17}{1+0.0{,}17} = \frac{-0{,}17}{1}$$

$$= -0{,}17$$

$$\boldsymbol{tg(180° - 10°) = -0{,}17}$$

8. tg(180° - 20°)

Solución:

$$tg(180° - 20°) = \frac{0 - 0,36}{1 + 0.0,36} = \frac{-0,36}{1}$$
$$= -0,36$$

$$\boldsymbol{tg(180° - 20°) = -0,36}$$

9. tg(180° - 50°)

Solución:

$$tg(180° - 50°) = \frac{0 - 1,19}{1 + 0.1,19} = \frac{-1,19}{1}$$
$$= -1,19$$

$$\boldsymbol{tg(180° - 50°) = -1,19}$$

10 tg(180° - 40°)

Solución:

$$tg(180° - 40°) = \frac{0 - 0,83}{1 + 0.0,83} = \frac{-0,83}{1}$$
$$= -0,83$$

$$\boldsymbol{tg(180° - 40°) = -0,83}$$